Optimization and Modeling at UW:
Fish, Cows, Sanctions and Energy

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How to enhance the impact of optimization in applications?

- Hire (and/or engage) people with breadth of, and complementary expertise - theory, algorithms, computation, applications
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- Hire (and/or engage) people with breadth of, and complementary expertise - theory, algorithms, computation, applications
- Key impact area: decision making in (environmentally) resource constrained problems
- Feature: shared resource that interacts with complex multi-user systems
- Enhance understanding of decision space, facilitate policy design and operational improvement
- Build **appropriate models**, fast enough solution for expert interaction, visualize results
Overview

- Anadromous fish migrate from the sea upstream into freshwater to spawn.
- Natural & man-made barriers break stream connectivity and prevent fish from penetrating deep into inland lakes and rivers.

There are over 235,000 identified barriers to migration in the Great Lakes Basin:
- Lake Michigan: >83% of tributaries inaccessible
- Lake Huron: >86% of tributaries inaccessible
- Lake Erie: >50% reduction of population size
• Barriers can be mitigated to allow for fish passage:
  ▪ Removal of dams, improved road crossings, fish passageways
• However, they are very expensive – Average costs for fixes:
  ▪ Dams: $100,000 - $650,000 each
  ▪ Others: $30,000 - $150,000 per project
• Limited funds necessitate ideal selection of projects
  ▪ Difficult to assess where funds should be used
  ▪ Country/State/County lines make appropriation difficult
• Increasing passability increases risk for the spread of invasive aquatic species (e.g. Sea Lamprey)
The Goal (Customer #1)

1. Provide an interactive tool to consolidate big-data sets across multiple departments (DNR, FWS, NFPP, etc) and visually display in a meaningful way.

2. *Utilize optimization to maximize efficiency in policy decisions and funds appropriations.

3. *Allow any user to dynamically solve a large range of models and scenarios without requiring background knowledge of optimization.

4. Provide means for certified users to update/validate data sets.
Data Visualization: http://www.greatlakesconnectivity.org/
The Data

For every Barrier $[J]$: 236,264
- Barrier ID – A unique string identifier
- Geographical Info – Nation, State, County, Lake Basin, Watershed
- Barrier Type – Dam or Road Passage
- Cost – Estimated cost to mitigate the barrier
- Root – If the barrier is the first in the stream (no downstream barriers)
- Downstream ID – Identifier of the downstream barrier

For every Fish Guild $[S]$: 36
- Invasive – If it is an invasive species or not

For every $[J \times S]$: 8,505,504
- Passability Rating – % Chance species can pass this barrier
- Upstream Habitat – Amount of usable habitat upstream of barrier
The Model

Objective:

\[
\max \sum_{j \in J} \sum_{s \in S \setminus \text{Inv}} v_{js} \cdot z_{js}
\]

Subject To:

\[
\sum_{j \in J} x_j \cdot c_j \leq B
\]

\[
z_{js} = \left(\bar{p}_{js} + \pi_{js} \cdot x_j\right) \cdot z_{ds}, \quad \forall j \in J, d \in D(j), s \in S
\]

\[
x_j \in \{0,1\}
\]

Where:

- \(v_{js} := \text{Upstream Habitat}\), \(\bar{p}_{js} := \text{Passability Rating}\), \(\pi_{js} := \text{Probability Increase (if mitigated)}\)
- \(c_j := \text{Cost of mitigation}\), \(B := \text{Total Available Budget}\)
- \(z_{js} := \text{Cumulative passability rating}\), \(D(j) := \text{Set of nodes downstream of } j\). Note: |D(j)| \leq 1.
- \(x_j := \text{Decision to Remove barrier } j\)
Smart Modelling - Linearization

\[ z_{js} = (\bar{p}_{js} + \pi_{js} \cdot x_j) \cdot z_{ds}, \quad \forall j \in J, \forall d \in D(j) \]

Use set of roots \((R \subset J)\):

\[ z_{rs} = \bar{p}_{rs} + \pi_{rs} \cdot x_r, \quad \forall r \in R, s \in S \]

Introduce new variable \(y_{js} = x_{js} \cdot z_{ds}\):

\[ z_{js} = \bar{p}_{js} \cdot z_{ds} + \pi_{js} \cdot y_{js}, \quad \forall j \in J \setminus R, s \in S \]

Add additional constraints:

\[ y_{js} \leq x_j, \quad \forall j \in J \setminus R, s \in S \]

\[ y_{js} \leq z_{ds}, \quad \forall j \in J \setminus R, s \in S \]
Basic \{0,1\} LP Model:

\[ \max \sum_{j \in J} \sum_{s \in S \setminus \text{Inv}} v_{js} \cdot z_{js} \]

Subject To:

\[ \sum_{j \in J} x_j \cdot c_j \leq B \]

\[
\begin{align*}
    z_{rs} &= \bar{p}_{rs} + \pi_{rs} \cdot x_r, & \forall r \in R, s \in S \\
    z_{js} &= \bar{p}_{js} \cdot z_{ds} + \pi_{js} \cdot y_{js}, & \forall j \in J \setminus R, s \in S \\
    y_{js} &\leq x_j, & \forall j \in J \setminus R, s \in S \\
    y_{js} &\leq z_{ds}, & \forall j \in J \setminus R, s \in S \\
    x_j &\in \{0,1\}, & \forall j \in J
\end{align*}
\]
Interactive Modelling

Allow user to:

- Select their range of influence (i.e. State, County, etc)
- Select mitigatable barriers using a broad range of criteria
- Manipulate Constraints
- Visualize Results

Let’s check it out! ($B = 10^7$)

- Minnesota : $3,458 – 6$s.
- Wisconsin: $19,854 –$ Timed Out!?
{0,1} Linear Programming is $NP$ – Complete!
- Solution time quickly becomes unpractical as problem size grows!
- Web tool requires fast processing to inform user.

Need to find methods to speed up solution time!
Could we take advantage of the unique structure of our data?
Pre-Processing

Disjoint Counties: Data Compression

- May desire collaboration between counties
- Downstream barriers effected by upstream decisions
  - Barriers in-between are irrelevant
  - Can be removed by smartly incorporating their data into other nodes!
Representative Species

- 36 total fish guilds – Many have very similar parameter data!
- Use QAP to separate guilds into ‘representative groups’
  - Smaller overall data set – improves speed of (relaxed) master solution

\[
\min \left\{ \sum_{g \in G} \sum_{i \in S} \sum_{j \in S} (d_{ij} \times z_{ijg}) \right\}
\]

Subject to:

\[
\sum_{g \in G} x_{sg} = 1, \quad \forall s \in S
\]

\[
z_{ijg} \leq x_{ig}, \quad \forall i, j \in S, g \in G
\]

\[
z_{ijg} + 1 \geq x_{ig} + x_{jg}, \quad \forall i, j \in S, g \in G
\]
Independent Streams?

Each root node corresponds to a completely independent tree!
Can solve separate, smaller MIP on each tree.

- However, budget constraint is global!
- *How do determine budget in each tree?*
The Goal (Customer #2)

Quickly and accurately create return-on-investment (ROI) curves for a wide-breadth of project scenarios.

- Each curve requires > 20 data points to cover all range of possible budgets!

Supplement base model with additional constraints:

- Ensure that available habitat for ALL species increases by specific amount
  - While still maximizing total habitat
- Prevent invasive species from gaining too much habitat.
Test Data Set: Lake Huron Basin

51,149 Barriers
36 Species
  - 2 Invasive Species

Model Size:
  - 1,934,421 rows
  - 1,274,454 columns
  - 753 discrete-columns
  - 4,896,386 non-zeroes
The Problem:

• \{0,1\} Linear Programming is \(\mathcal{NP} – \text{Complete}\)!
• Our Data Set is extremely large.
• Solution times grow exponentially with budget [CPLEX, WID Clusters]:
  - \(B = 10^6\): 8211 s (Gap = 0%)
  - \(B = 10^7\): 2132 s (Gap = 0%)
  - \(B = 10^8\): >4 days (Gap = 1%)
  - \(B = 5 * 10^8\): >4 days (Gap = 10%)
• Customer desires ROI Curve generation, requiring data points over the entire range of budgets and different scenarios!
• Solution time is unpractical for dynamic web-app modelling!
As we can see, we are able to obtain reasonable solutions for most budgets in less than 10 minutes!

<table>
<thead>
<tr>
<th>Budget ($)</th>
<th>Sol Time (s)</th>
<th>Gap (%)</th>
<th>Sol Time for Best (s)</th>
<th>% Speedup</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6$</td>
<td>573</td>
<td>0.53</td>
<td>8211</td>
<td>1,333 %</td>
</tr>
<tr>
<td>$10^7$</td>
<td>668</td>
<td>0.88</td>
<td>2132</td>
<td>219 %</td>
</tr>
<tr>
<td>$10^8$</td>
<td>2431</td>
<td>1.31</td>
<td>&gt; 4 days</td>
<td>14,116 %</td>
</tr>
</tbody>
</table>
Already Impactful!

- Researchers at UW Limnology believe(d) that invasives constraint is vital to amount of attainable habitat.
  - Large amounts of research conducted to identify spread threats
  - Investing research $ into improving mitigation/treatment techniques
    - Pheromones, lampricide, traps, low-head barriers, etc
- ROI Curves show otherwise!
- Either...
  - We’ve discovered a flaw in current theories on invasive species spreading
  - Or... (More Likely), a flaw in the data set.
Biomass Research and Development Initiative (BRDI)

- Whole farm (complex interacting) mathematical model
- Long term sustainable (environment and financial)
- Economic/Logistic Optimization, taking into account phosphorus runoff, other environmental restrictions
- Incorporates data analytics (e.g. SNAP+)
- New insights to operate system efficiently, how to enforce much stricter environmental constraints using blend of rotations, NMP and separations
- Large (mixed integer) optimization
Optimal Sanctions (Boehringer/F./Rutherford)

- GTAP global production/trade database: 113 countries, 57 goods, 5 factors
- Coalition members strategically choose trade taxes to minimize Russian welfare
- Russia chooses trade taxes to maximize Russian welfare in response
- Nash equilibrium

Resulting equilibrium with no regrets (coalition), maximize damage, side payments
Security-constrained Economic Dispatch (SCED)

\[
\begin{align*}
\min_{u, x_0, \ldots, x_k} & \quad c^T u + \rho(u) \\
\text{s.t.} & \quad 0 \leq u \leq \bar{u} \\
& \quad g_0(x_0, u) = 0 \\
& \quad -\bar{x} \leq x_0 \leq \bar{x} \\
& \quad g_k(x_k, u) = 0, \quad k = 1, \ldots, K \\
& \quad -\bar{x} \leq x_k \leq \bar{x}, \quad k = 1, \ldots, K
\end{align*}
\]

- Total cost
- GEN capacity const.
- Base-case network eqn.
- Base-case flow limit
- Ctgcy network eqn.
- Ctgcy flow limit

- Base-case topology \(g_0\) and line flow \(x_0\)
- If \(k\)-th line fails, line flow jumps to \(x_k : g_k = 0\)
- Ensure \(x_k\) in bounds \(\forall k\)
Model structure

- Corrective actions are not modeled in ISO’s dispatch software (deemed unsolvable!)
- We **model** the *multi-period* corrective rescheduling in SCED; solutions much better quality
- **Enhance** the Benders’ **algorithm** to solve the problem faster
- **Achieve** about $50 \times$ **speedup** compared to traditional approaches

**Figure**: Sparsity structure of the Jacobian matrix of a 6-bus case, considering 3 contingencies and 3 post-contingency checkpoints.
Conclusions

- Optimization guides the development of complex interaction processes within application domains.
- Combination of models provides effective decision tool at multiple scales.
- Policy implications addressable using MOPEC.
- Problems solved by combination of domain expertise, modeling prowess, good theory/algorithms and efficient implementations all facets needed.
- Many new settings available for deployment; need for more theoretic and algorithmic enhancements.